4. The Impact of ITemp Applied to a Mobile Services CDMA Network

This section considers the possibility of applying the ITemp concept to a network providing mobile services using CDMA technology. As discussed below, it is clear that the forward link (base to mobile, or downlink) is not a practical candidate for ITemp. Although ITemp might in theory be applied to the reverse link (uplink), there are significant practical challenges with implementation, and as shown by the analysis below, even if those challenges are ignored and perfect implementation is assumed, the net impact on the overall efficiency of spectrum utilization is negative. In other words, at least in the case of mobile CDMA networks, the ITemp concept is spectrally inefficient, because the value of the unlicensed capacity gained is less than that of the licensed capacity lost due to the added interference.

4.1 Forward Link

The CDMA forward link transmits overhead channels (pilot, sync, paging) and traffic channels to the mobiles. At any given time, most mobiles in a network are idle (receiving the overhead channels but not connected *via* a traffic channel) and are therefore not transmitting. Battery life is typically many days in the idle state for all digital mobile radio technologies due to the receiver sleep cycle (the receiver periodically wakes up to acquire the overhead channels). By their nature, the locations of the mobiles are unknown and changing constantly, and there can be no guaranteed separation between a mobile and an unlicensed transmitter. Therefore, to be effective, ITemp interference monitoring would have to be done within the mobile unit itself.

Embedding the monitoring and feedback functionality within licensed mobile devices themselves does not seem practical. Unlicensed devices would need to receive feedback signals from a number of mobiles. Organizing the transmission of these feedback signals so they would not mutually interfere and so the unlicensed devices could decode them all would likely require sophisticated scheduling, which would impact the entire mobile radio system. The unlicensed device would need to calculate its transmit power based on the most vulnerable mobile, where vulnerability would depend on proximity of the mobile to the unlicensed device as well as the interference already experienced by the mobile. The proximity of the mobile presumably would be calculated based on the strength of the received feedback signal.

Thus, a mobile would need to (a) determine the total received power from the unlicensed devices, in the presence of the much stronger CDMA downlink signal, and (b) transmit (fairly often) the feedback signal conveying the result of this measurement. The impact of these requirements on the cost and battery life of the mobiles would be severe. Moreover, a dedicated frequency would be needed for the feedback signal, further increasing the cost and complexity of the mobile unit.

In sum, it does not appear practical, even in concept, to apply ITemp to the forward link (downlink) of mobile services.

4.2 Reverse Link

In theory, the ITemp concept might be applied to the uplink of a mobile system, since the base stations are always transmitting, and it might be possible to infer the path loss between an ITemp device and the base station receiver by measuring the power on the broadcast downlink channels, assuming that the transmitted power (into the base station antenna terminals) is somehow known. This section develops a mathematical model for the impact of such an implementation on the CDMA uplink, showing the relationship between $\Delta T/T$ and the uplink capacity reduction of the CDMA system. Following that, the allowable transmit power for the unlicensed devices is shown as a function of distance from the CDMA base station, and implementation issues are discussed.

4.2.1 SINR and Jamming Margin

If E_b is the received energy per bit on a particular uplink channel, and N_0 and I_0 are the power spectral density (watts/Hz) of the thermal noise and total interference, respectively, then the signal to interference plus noise ratio (SINR) is $E_b/(N_0 + I_0)$, which must meet or exceed some threshold Γ for the channel to meet its frame error rate (FER) objective. That is,

$$\frac{E_b}{N_0 + I_0} \geq \Gamma, \tag{43}$$

where the threshold Γ in general depends on a number of factors, including the multipath delay spread (which determines the RAKE diversity combining gain), interleaving depth, fade rate, type of channel coding, target FER, and the accuracy of the closed-loop (fast) power control.

If the channel intermediate-frequency (IF) channel bandwidth is *W* Hz and the data rate is *R* bps, the “spreading gain” (or “processing gain”) is W/R. Letting *C* represent the received carrier (desired signal) power, and *N* and *I* represent the noise and interference power, respectively, at the receiver, the relationships $E_b = C/R$, $N = WN_0$, and $I = WI_0$ lead to the identity:

$$\frac{E_b}{N_0 + I_0} = \frac{W}{R}\frac{C}{N + I}. \tag{44}$$

Defining the “jamming margin” as

$$M = \frac{W/R}{\Gamma} \tag{45}$$

and combining (43) and (44) gives:

$$\frac{C}{N+I} \geq \frac{1}{M}. \tag{46}$$

4.2.2 Basic Uplink Capacity Relationships

There are assumed to be J terminals in the cell transmitting on the uplink. The desired signal power received from the j^{th} terminal is denoted C_j. The total power received from these J terminals is

$$I_{in} = \sum_{j=1}^{J} C_j \tag{47}$$

In addition, the base station receiver sees interference power from other cells of the same system, denoted I_{oc} as well as its own thermal noise power N. In case of interest here, the uplink (base station) receiver also sees some permissible level of additive external interference power, denoted I_{ext}. The total noise plus interference at the receiver therefore is

$I_{TOT} = N + I_{in} + I_{oc} + I_{ext}$ (The interference plus noise seen by the receiver component associated with the j^{th} terminal is $I_{TOT} - C_j$. Therefore, from (46),

$$\frac{C_j}{I_{TOT} - C_j} \geq \frac{1}{M_j} \tag{48}$$

where $M_j = \dfrac{W/R_j}{\Gamma_j}$ is the jamming margin for the j^{th} terminal, and R_j and Γ_j are the associated data rate and minimum SINR, respectively. Rearranging (48) and assuming equality gives:

$$C_j = \frac{I_{TOT}}{M_j + 1} \tag{49}$$

Hence,

$$I_{in} = \sum_j C_j = I_{TOT} \sum_j \frac{1}{M_j + 1} \tag{50}$$

To simplify notation in the analysis that follows, it is useful to define

$$\begin{aligned} \Lambda_j &= \frac{1}{M_j + 1} \\ \Lambda &\equiv \sum_j \Lambda_j \end{aligned} \tag{51}$$

so

$$I_{in} = \Lambda I_{TOT} \tag{52}$$

The parameter Λ is a good measure of the total load carried by the uplink. To see this, assume that $M_j >> 1, \forall j$ (true for low-rate services such as speech; for IS-95 rate set 1, the spreading gain is 21 dB and the required SINR is about 7 dB, giving a jamming margin on the order of 14 dB, or a factor of 25), in which case

$$\Lambda \cong \frac{\Gamma}{W} \sum_j R_j = \frac{\Gamma}{W} R_{TOT} \tag{53}$$

If $\Gamma_j = \Gamma, \forall j$, then

$$\Lambda \cong \frac{\Gamma}{W} \sum_j R_j = \frac{\Gamma}{W} R_{TOT} \tag{54}$$

where $R_{TOT} = \sum_j R_j$ is the total uplink throughput for the cell. Therefore, Λ will be referred to as the “load” carried by the cell uplink. The larger Λ is, the greater the total throughput, given the bandwidth W and the SINR thresholds $\{\Gamma_j\}$ (which in general are not equal). Maximizing Λ corresponds to maximizing uplink cell capacity.

In a uniformly-loaded system, the other-cell interference is proportional to the in-cell interference; that is $I_{oc} = mI_{in}$. For terrestrial systems, m is typically on the order of 0.4 to 0.6, depending on propagation. Using this relationship, along with (and (52) results in:

$$\frac{I_{TOT}}{N} = 1 + \Lambda \frac{I_{TOT}}{N}(1+m) + \frac{I_{ext}}{N} \tag{55}$$

or

$$\frac{I_{TOT}}{N} = \frac{1}{1-\Lambda(1+m)}\left(1+\frac{I_{ext}}{N}\right) \tag{56}$$

For a terrestrial cellular or PCS system with an exclusive allocation, $I_{ext} = 0$, and (56) leads to the well-known CDMA load curve, shown in Figure 18.

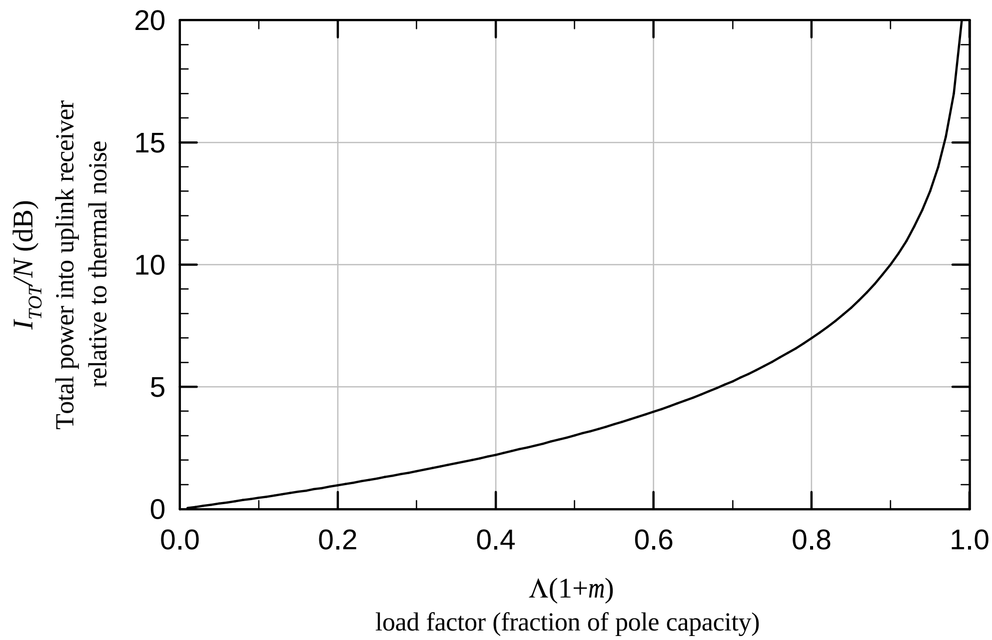

Figure 18: *CDMA uplink load curve*

The "pole capacity" corresponds to $\Lambda(1+m)=1$, at which point I_{TOT}/N is unbounded. CDMA systems impose an upper bound on I_{TOT}/N to limit the required dynamic range on the uplink receiver as well as the required terminal transmit power. A reasonable limit would be on the order of 6 dB, corresponding to $\Lambda(1+m)=0.75$. This limit is enforced by the admission control mechanism, and determines the maximum capacity of the uplink.

4.2.3 Uplink Capacity Reduction due to External Interference

The addition of the other-system interference I_{ext} clearly reduces the available uplink capacity. To quantify the capacity reduction, assume that $\Phi = \left(I_{TOT}/N\right)_{\max}$ is the system-specified upper limit. Without the other-system interference, the uplink capacity is

$$\Lambda_0 = \frac{1}{1+m}\left(1-\frac{1}{\Phi}\right). \tag{57}$$

Adding the other-system interference reduces the capacity to

$$\Lambda = \frac{1}{1+m}\left[1-\frac{1}{\Phi}\left(1+\frac{I_{ext}}{N}\right)\right], \tag{58}$$

and the capacity reduction is

$$\Delta\Lambda = \Lambda_0 - \Lambda = \frac{1}{1+m}\frac{1}{\Phi}\frac{I_{ext}}{N}. \tag{59}$$

As a fraction of the stand-alone capacity, the reduction is

$$\frac{\Delta\Lambda}{\Lambda_0} = \frac{I_{ext}/N}{\Phi - 1}. \tag{60}$$

In terms of the interference temperature ΔT:

$$\frac{\Delta\Lambda}{\Lambda_0} = \frac{\Delta T/T}{\Phi - 1} \tag{61}$$

Figure 19 shows the fractional capacity reduction in percent vs. $\Delta T/T$, also in percent, assuming $\Phi = 4$ (6 dB). In that case, the relationship is simply

$$\frac{\Delta\Lambda}{\Lambda_0} = \frac{1}{3}\frac{\Delta T}{T}. \tag{62}$$

Figure 20 shows Λ/Λ_0 vs. I_{ext}/N in dB.

Note that if the external interference is three times the noise, capacity drops to zero (the interference has consumed the entire reverse link received power budget).

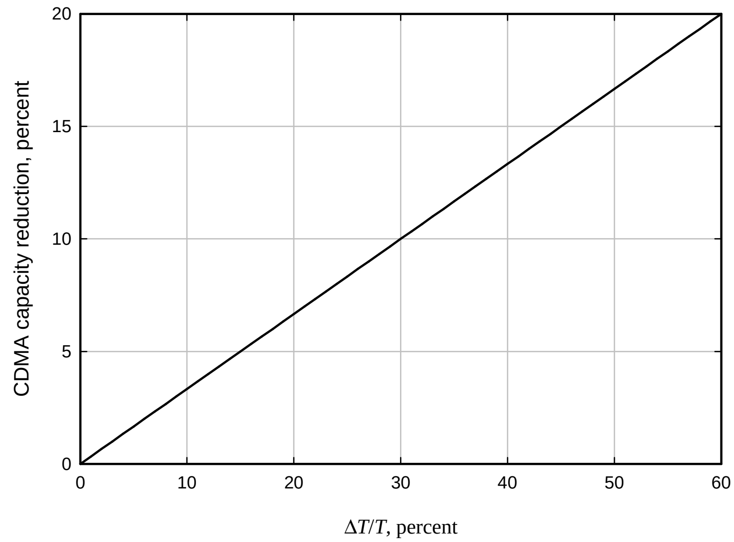

Figure 19: *CDMA uplink capacity reduction express as interference temperature relative to baseline noise temperature.*

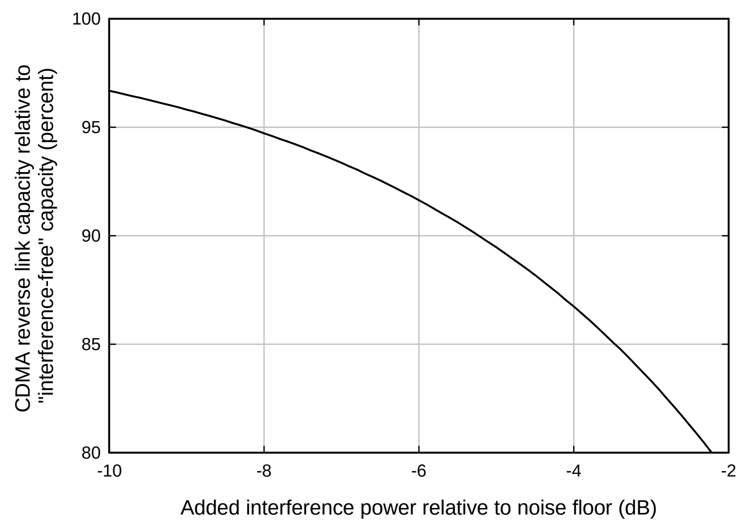

Figure 20: *CDMA reverse link capacity reduction due to external interference.*

The receiver noise is

$$N = -174 + 10\log W + F \quad \text{dBm} \tag{63}$$

where F is the receiver noise floor in dB. Assuming that $F = 5\,\text{dB}$ and $W = 1.25\,\text{MHz}$, then $N = -108\,\text{dBm}$. Figure 21 shows the reverse link capacity reduction $\Delta\Lambda/\Lambda_0$ (as a percentage) from (60), assuming this value of N, and also assuming as before that $\Phi = 4$. Note that for $\Delta T/T = 0.06$, corresponding to a 2% capacity reduction, $I_{ext} = -120\,\text{dBm}$.

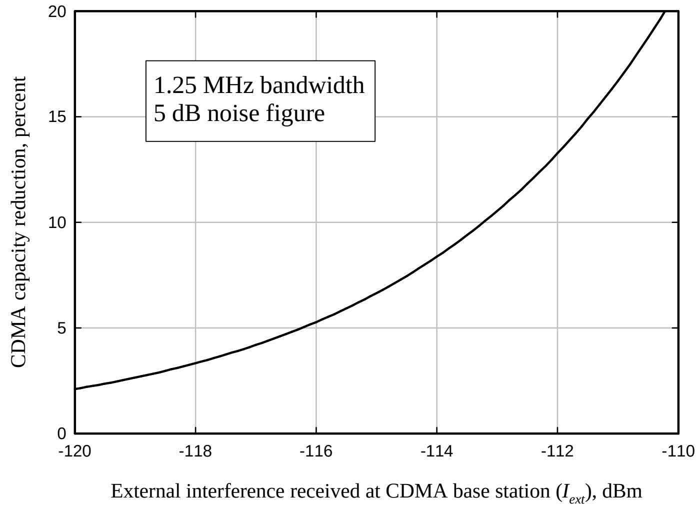

Figure 21: *CDMA uplink capacity reduction vs. the level of external interference at the base station receiver.*

4.2.4 Uplink Throughput Reduction

The reduction in uplink throughput can also be computed. From (51),

$$\Lambda = \sum_j \frac{1}{M_j + 1} \tag{64}$$

Assume that all links have the same data rate R and SINR requirement Γ, and hence the same jamming margin M. The total uplink throughput per sector is then

$$R_{TOT} = R\Lambda(M + 1) \tag{65}$$

Note that since $M = W/\Gamma R$,

$$\begin{aligned}\Lambda &= \frac{\Gamma}{W} R_{TOT}\left(\frac{1}{1+1/M}\right) \\ &\cong \frac{\Gamma}{W} R_{TOT} \qquad M >> 1\end{aligned} \tag{66}$$

which agrees with (53).

From (65), the lost throughput relative to the base is

$$\frac{\Delta R_{TOT}}{R_{TOT,0}} = \frac{\Delta\Lambda_0}{\Lambda_0} = \frac{\Delta T/T}{\Phi-1} \tag{67}$$

Hence, the lost throughput per sector is:

$$\begin{aligned}\Delta R_{TOT} &= \frac{\Delta T/T}{\Phi-1} \cdot R\Lambda_0 (M+1) \\ &= \frac{\Delta T/T}{\Phi(1+m)} \cdot R \cdot (M+1)\end{aligned} \tag{68}$$

4.2.5 EIRP of Unlicensed Devices

In principle, an unlicensed device could, by monitoring the strength of the CMDA downlink pilot, infer the path loss and compute the transmit power level that would result in a given received power level at the CDMA base station. If L_p is the path loss in dB, P_{pilot} is the pilot transmit power (applied to the base station antenna terminals), C_{pilot} is the pilot power received at the antenna terminals of the unlicensed device, $I_{\max}$ is the maximum level of external interference that the unlicensed device is allowed to generate at the base station antenna terminals, and G_b and G_{ul} are the respective antenna gains of the CDMA base station and the unlicensed device, then:

$$L_p - G_b - G_{ul} = P_{pilot} - C_{pilot} \tag{69}$$

and the maximum allowed transmit power level for the unlicensed device is

$$\begin{aligned}P_{ul} &= I_{\max} + L_p - G_b - G_{ul} \\ &= P_{pilot} + I_{\max} - C_{pilot}\end{aligned}. \tag{70}$$

That is, if P_{pilot} and $I_{\max}$ are known to the unlicensed device, and if the unlicensed device has the capability to measure C_{pilot}, then it may be able to compute P_{ul}.

A propagation model is needed to estimate the unlicensed transmit levels that this approach would allow. Figure 22 shows the median path loss vs. distance from the base station according to the Hata model as modified by COST 231 for frequencies in the 1.5-2 GHz range.

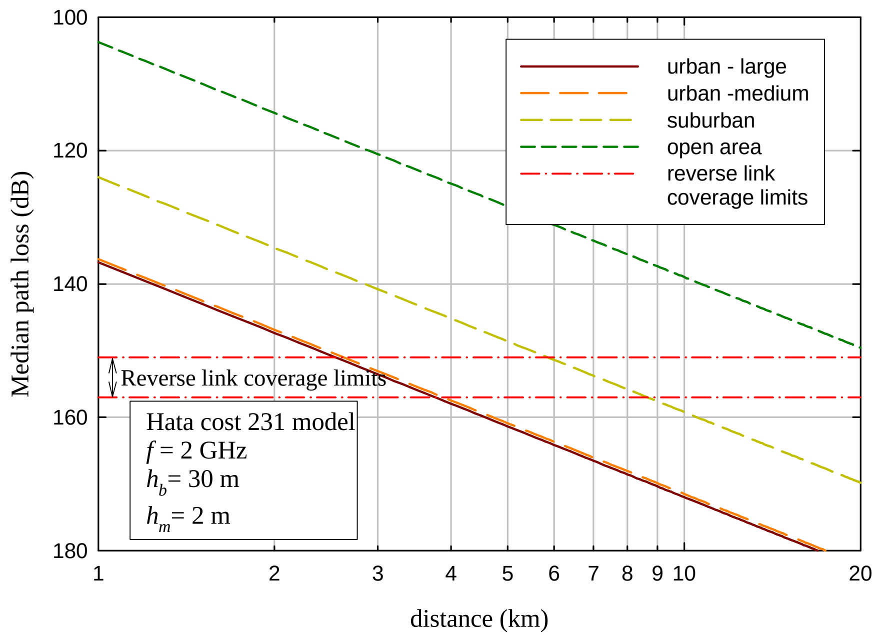

Figure 22: *Median path loss vs. distance from base station, COST 231 Hata model.*

The reverse link coverage limits shown were calculated assuming a maximum handset transmit power of 23 dBm, a base station antenna gain of 12 dBi, and a reverse link E_b/N_0 requirement of 7 dB, giving an SINR requirement of –14 dB. With the cell unloaded, the only impairment is thermal noise, assumed –108 dBm, and the required received signal is –122 dBm. The maximum path loss is therefore 23 + 122 + 12 = 157 dB, which is the lower line. With full loading, the interference plus noise is –102 dBm and the maximum path loss is 151 dB, corresponding to the upper line.

The unlicensed device EIRP can be computed from (70) as:

$$EIRP_{ul} = P_{ul} + G_{ul} = I_{\max} + L_p - G_b \tag{71}$$

As an example, consider the suburban case and a distance of 3 km, for which the path loss shown is about 140 dB. Assume that the antenna gain of the CDMA base station is

12 dBi. To generate a received level of –120 dBm at the base station antenna terminals, the unlicensed device can transmit 8 dBm EIRP.

Of course, this analysis applies only to a single unlicensed device. If there are *K* unlicensed devices active, and each has calculated its transmit power to result in –120 dBm interference at the CDMA base, then the total interference into the base receiver is

$$I_{ext} = -120 + 10\log K \ \ \text{dBm} \tag{72}$$

If the unlicensed devices are independently measuring the downlink pilot power and computing their allowed transmit power levels, then the potential clearly exists for exceeding the interference temperature threshold.

There seem to be two solutions to this problem: (1) allow for a certain number of unlicensed devices and allocate interference power equally among them; and (2) provide some sort of feedback mechanism that would limit the aggregate power from the unlicensed devices as seen at the CDMA base station.

As an example of the first approach, assume that per-device interference is based on $K = 100$, which case,

$$P_{ul} = P_{pilot} + I_{\max} - C_{pilot} - 20 \ \ \text{dBm} \tag{73}$$

and EIRP per unlicensed transmitter is:

$$EIRP_{ul} = P_{ul} + G_{ul} = I_{\max} + L_p - G_b - 20 \ \ \text{dBm} \tag{74}$$

as shown in Figure 23. Assuming that the CDMA cells are arranged to provide coverage at full loading (which in this case is assumed to be 75% of pole capacity), the maximum EIRP per unlicensed device, at the edge of the cell, is –1 dBm or about 0.8 mW. Note that this is independent of the assumed propagation model; it depends on the path loss, but not on the distance that corresponds to that path loss.

In a multi-cell system, the unlicensed device would be able to use the maximum power level (e.g., –1 dBm EIRP) on the cell boundaries. Toward the interior of the cell, the power would be reduced as indicated in Figure 23.

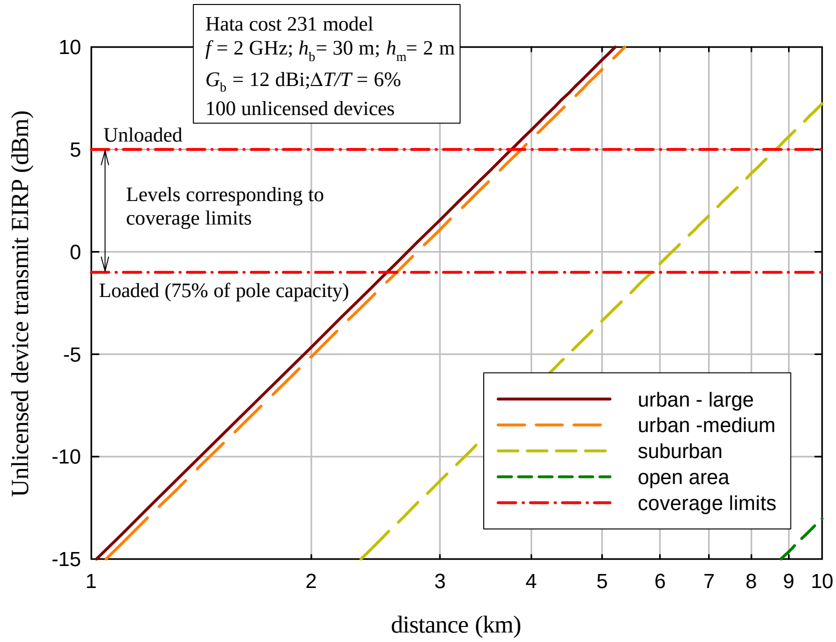

Figure 23: *Maximum transmitted EIRP (dBm) for 100 unlicensed devices to give* $\Delta T/T = 0.06$ *on a CDMA PCS uplink.*

4.2.6 Implementation Issues

A major problem with this first approach (limiting the number of unlicensed devices in an area) is that an *a priori* estimate of the number of unlicensed devices per cell must be used to set the transmit power limits. This problem could be solved (in principle) with a feedback mechanism, whereby the total interference power from the unlicensed devices at the base station is monitored. If the total interference exceeds the predetermined threshold, a “reduce power” signal could be broadcast to all of the unlicensed devices. The power reduction might be small (e.g., 0.5 dB). This is similar in concept to the power control used in the CDMA system itself, except that it operates in a broadcast mode rather than being directed to a specific device.

This “broadcast power control” (BPC) could be implemented in different ways. One possibility would be to have the CDMA receiver monitor its noise level, and if the external interference exceeds a threshold, cause the signal to be transmitted on the CDMA downlink (*e.g.*, on a paging channel). However, there appear to be two problems with this approach. First, the CDMA system would need to be modified to include the required downlink message. Second, it is not clear how the CDMA base station receiver would be able to distinguish the external interference from other-cell reverse link interference.

An alternative would be to use a dedicated receiver, collocated with the CDMA receiver, to sense the external interference, and a dedicated transmitter to issue to the power control command. This would require installation of an antenna with a pattern matched to that of the CDMA base transceiver. In addition, the receiver would need to be able to separate the CDMA signals from the unlicensed signals – that is, it would need to be matched to the waveform transmitted by the unlicensed devices. Given that the aggregate power from the unlicensed devices would be on the order of 18 dB below the power received on the CDMA uplink, this would represent a fairly challenging signal processing problem. One possible solution would be to have a component to the unlicensed transmission that is outside the band of the CMRS uplink. The total power of the received out-of-band component could be used to compute the aggregate in-band unlicensed power. Unfortunately, this approach requires some dedicated spectrum for the out-of-band signal component. It would also require that the ITemp system obtain tower space for the monitoring antenna and the associated equipment.

4.2.7 Summary

This section has provided the mathematical framework for analyzing the impact of ITemp-based unlicensed devices sharing an uplink frequency with a CDMA CMRS system and has provided a relationship between the added interference and the resulting capacity degradation of the CDMA uplink. Some of the implementation problems have also been outlined, and it is clear that there would be significant practical challenges to implementing the approach described here.

Notwithstanding these implementation challenges, it is worthwhile to assume a perfect implementation (without regard to how it might be realized), and examine the tradeoff between what is gained in terms of unlicensed capability versus the loss of licensed capacity. This is the topic of the next section.

5. Cost/Benefit Analysis for ITemp Sharing of the CDMA Uplink

It is clear from the previous section that controlling the aggregate power output of the spectrum-sharing unlicensed users to maintain $\Delta T/T$ below some threshold is a daunting problem. However, this section assumes that such control can be somehow accomplished. With that assumption, this section examines the potential spectrum efficiency associated with implementing the ITemp concept in the CDMA uplink. The analysis suggests that the ITemp concept would not be spectrally efficient in this situation.

5.1 Carrier-to-Interference Ratio as Seen by the Unlicensed Receivers

If the path loss between the handset and the base station is L_h, then the handset transmit EIRP is

$$EIRP_h = I_{TOT} - 10\log(M+1) + L_h - G_b \qquad \text{dBm} \tag{75}$$

where $M = (W/R)/\Gamma$ is the jamming margin discussed earlier, and I_{TOT} is the total noise plus interference (in cell, other cell, and external) at the base station receiver.

If the total unlicensed device power into the CDMA base station could be somehow perfectly controlled such that it is $I_{\max}$, then if there are K_u active unlicensed radios in the CDMA sector, the EIRP for an unlicensed device with path loss L_u is

$$EIRP_u = I_{\max} + L_u - G_b - 10\log K - ICF_{u,c} \text{ dBm} \tag{76}$$

where $ICF_{u,c}$ is the interference correction factor for interference from the unlicensed device into the CDMA receiver. This assumes that the $\Delta T/T$ "interference budget" is apportioned equally among the K unlicensed devices.[6]

Consider an unlicensed transmitter attempting to communicate with its companion receiver, where the transmitter and receiver are separated by a distance d_u and the associated path loss is $L(d_u)$. An active CDMA handset interference is a distance d_h

[6] A cooperating group of unlicensed devices may choose to divide their power budget unequally; this is discussed later.

from the unlicensed receiver, and the path loss between them is $L(d_h)$. The desired signal power received by the unlicensed receiver is

$$C_u = EIRP_u - L(d_u) + G_u \quad \text{dBm} \tag{77}$$

where G_u is the gain of the unlicensed receive antenna.

The interference at the unlicensed receiver due to the CDMA handset a distance d_h away is:

$$I_h = EIRP_h - L(d_h) + G_u + ICF_{c,u} \quad \text{dBm} \tag{78}$$

where $ICF_{c,u}$ is the interference correction factor when the CDMA waveform is the interfering signal and the unlicensed receiver is the victim.

The carrier-to-interference ratio (CIR) at the unlicensed receiver is:

$$\begin{aligned} C_u - I_h = {} & [I_{\max} - I_{TOT} + 10\log(M+1)] + (L_u - L_h) - 10\log K_u \\ & -(ICF_{u,c} + ICF_{c,u}) + L(d_h) - L(d_u) \end{aligned} \quad \text{dB} \tag{79}$$

Aggregating parameters as

$$10\log a = [I_{\max} - I_{TOT} + 10\log(M+1)] + (L_u - L_h) - (ICF_{u,c} + ICF_{c,u}) \quad \text{dB} \tag{80}$$

gives the CIR as an absolute ratio as

$$\frac{C_u}{I_h} = \frac{a}{K_u} \frac{L(d_h)}{L(d_u)}. \tag{81}$$

If W_u and R_u are the respective bandwidth and data rate of the unlicensed transmitter, then the E_b/N_0 for the unlicensed receiver is

$$\left(\frac{E_b}{N_0}\right)_u = \frac{W_u}{R_u}\frac{C_u}{I_h} = \frac{W_u}{R_u}\frac{a}{K_u}\frac{L(d_h)}{L(d_u)} \tag{82}$$

5.2 Unlicensed Receiver Outage Probability

Of interest is the “outage” probability for the unlicensed receiver, which is defined here as the probability that $(E_b/N_0)_u$ is below some critical threshold Γ_u. That is,

$$P_{out} = \Pr\left[\left(\frac{E_b}{N_0}\right)_u < \Gamma_u\right] = \Pr\left[L(d_h) < \Gamma_u \frac{R_u K_u L(d_u)}{W_u a}\right] \tag{83}$$

Since $L(d_h)$ is a monotonically increasing function of d_h, the outage probability can be expressed as:

$$P_{out} = \Pr(d_h < d_0) \tag{84}$$

where

$$L(d_0) = \Gamma_u \frac{R_u K_u L(d_u)}{W_u a}. \tag{85}$$

Note that this outage definition does not account for aggregate interference from multiple CDMA handsets, and therefore the analysis here is somewhat optimistic.

The CDMA handsets can be regarded as randomly-located with respect to the unlicensed receiver. Assuming that the average density of active handsets is r_h, then the average number of handsets within some area *A* is rA. If the handsets are uniformly randomly distributed (a two-dimensional Poisson point process), then the probability that there are *j* handsets within some area *A* is

$$p_j = e^{-r_h A} \frac{(r_h A)^j}{j!} \tag{86}$$

so the probability that there are no handsets within area *A* is $e^{-r_h A}$. Therefore, the probability that there is at least one active CDMA handset within distance d_0 of the unlicensed receiver is:

$$P_{out} = \Pr(d_h < d_0) = 1 - e^{-p d_0^2 r_h} \tag{87}$$

For $P_{out} << 1$ (corresponding to high link reliability),

$$P_{out} \cong \pi d_0^2 \rho_h , \quad P_{out} << 1 \tag{88}$$

If there are K_h active handsets in the cell and the cell radius is r_c, then $\rho_h = K_h / \pi r_c^2$. Thus, for low outage,

$$d_0 \cong r_c \sqrt{\frac{P_{out}}{K_h}} \qquad P_{out} << 1 \tag{89}$$

and in general,

$$d_0 = r_c \sqrt{\frac{-\ln(1 - P_{out})}{K_h}} \ . \tag{90}$$

The capacity of a 3-sector 1xRTT cell is about 80 Erlangs at about a 2% blocking level (an average of 80 simultaneous calls), or $K_h = 40$. For a 2% outage probability for the unlicensed link, $d_0 \cong 0.022 r_c$. In an urban environment, $r_c \cong 2.5$ km, and in a suburban environment, $r_c \cong 6$ km (see Figure 22). Thus, d_0 is roughly 40 meters and 96 meters in these environments, respectively.

5.3 Local Propagation and Unlicensed Rate vs. Range

To calculate d_u, a propagation model is needed for the path loss between the unlicensed transmitter and receiver, and between the CDMA handset and the unlicensed receiver. Since the respective distances are relatively small and the antenna heights are likely to be low (e.g., 2 meters), the Hata model does not apply.

A reasonable model for a transmitter-receiver pair that are in close proximity is the "plane earth" or "smooth earth" model, in which there is a direct (free space) path between the transmitting and receiving antennas, as well as a ground-reflected path. Figure 24 shows the plane earth path loss for a frequency of 850 MHz, as well as free-space propagation curve and the plane-earth asymptote, which applies if the transmitter-receiver separation is large compare to the antenna elevations. Figure 25 shows the same curves for a frequency of 2 GHz. In both cases the transmitting and receiving antennas are assumed to be 2m above the earth's surface.

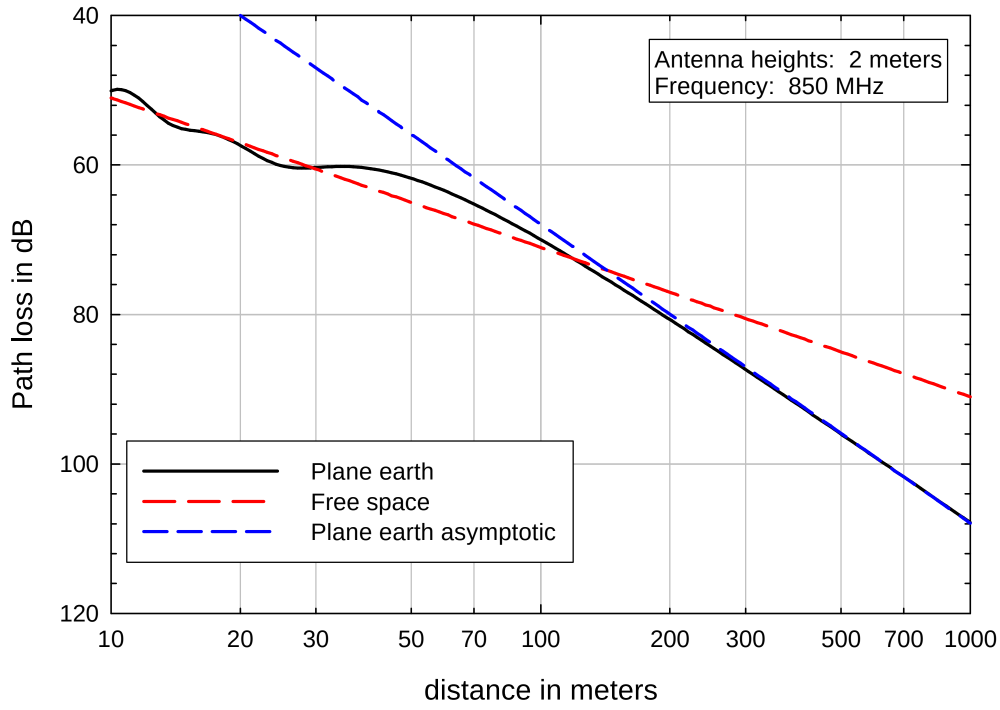

Figure 24: *Plane earth and free space path losses for 850 MHz and 2m elevations.*

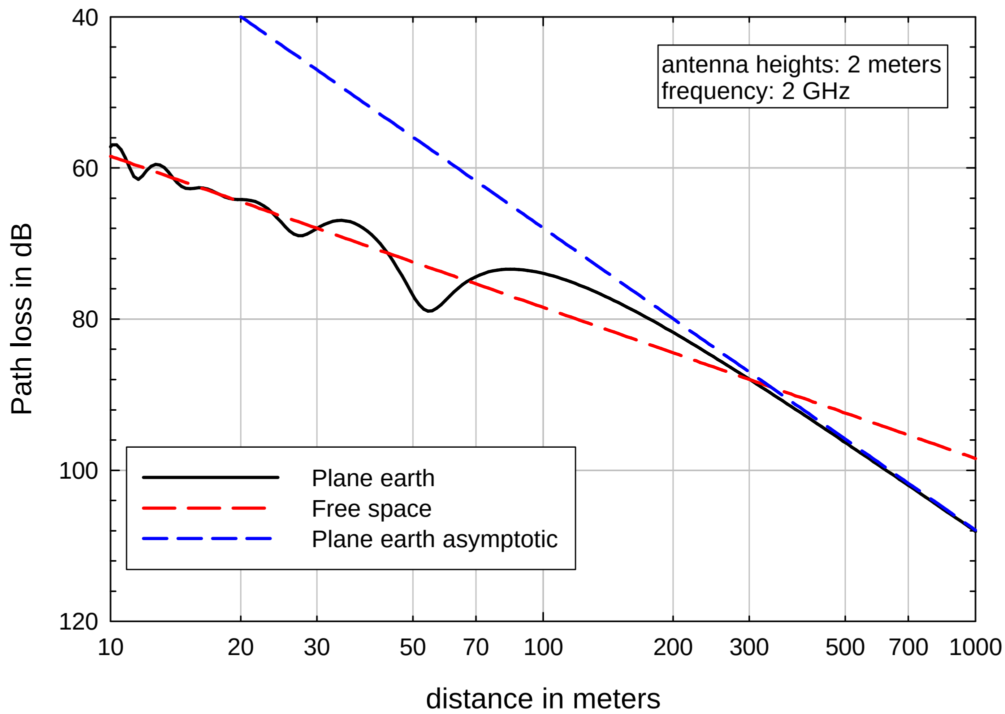

Figure 25: *Plane earth and free space path losses for 2 GHz and 2m elevations.*

As can be seen, for small distances, the path loss roughly follows the free space line, but with fluctuations due to changes in the relative phase between the direct and reflected components of the *E*-field at the receive antenna. Past the distance at which the free space and plane earth approximation lines cross (the “breakpoint” distance denoted d_{brk} here), the path loss closely follows the plane earth approximation, which is

$$L_4 = \frac{d^4}{(h_t h_r)^2} \tag{91}$$

where h_t and h_r are the transmit and receive antenna heights. The free space path loss is

$$L_{fs} = \left(\frac{4pd}{l}\right)^2 = \left(\frac{pdf_{MHz}}{75}\right)^2 \tag{92}$$

where l is the wavelength. These two are equal at the break point distance:

$$d_{brk} = h_t h_r \cdot f_{MHz} \cdot \frac{p}{75} \text{ m} \tag{93}$$

For $d < d_{brk}$, the path loss can be approximated as free space. Therefore, if $d_0 < d_{brk}$ and $d_u < d_{brk}$, then

$$\frac{L(d_u)}{L(d_0)} = \left(\frac{d_u}{d_0}\right)^2 \qquad d_0 < d_{brk}, \quad d_u < d_{brk} \tag{94}$$

and for $P_{out} << 1$,

$$d_u = d_0 \sqrt{a \frac{W_u}{\Gamma_u R_u K_u}} \tag{95}$$

As an example, assume that $ICF_{u,c} = 1$ and $ICF_{c,u} = 1$ (the CDMA and unlicensed signals appear noise-like to each other and have the same bandwidth), and $L_u = L_h$ (the CDMA handset and the unlicensed receiver have the same path loss to the CDMA base station). Based on the values for d_0 calculated above, this will be approximately true. Finally, let $I_{\max}/N = \Delta T/T = 0.06$, $I_{TOT}/N = 4$ (6 dB), and $M = 25$ (17 dB). With these values, $a = 0.38$.

Assume that $W_u / R_u K_u = 1$; that is, each unlicensed device is operating at 1 bps/Hz. Assuming that $\Gamma_u = 10\ \text{dB}$, then $d_u \cong 0.2 d_0$, or about $11 / \sqrt{K_u}$ meters in an urban environment and $26 / \sqrt{K_u}$ meters in a suburban environment. Thus, if there are 10 unlicensed devices operating within the sector, each at a modulation efficiency of 1 bps/Hz, and the degradation of the CDMA uplink capacity is limited to 2%, then for a 2% unlicensed outage level, the unlicensed devices will have a range of about 3.5 meters in an urban environment and 8.2 meters in a suburban environment. This compares to ranges of several hundred meters achievable by unlicensed devices operating in the ISM band.

While this example applies only to a single set of assumptions, those assumptions are not unreasonable. The results suggest that the operating range of ITemp-based unlicensed systems would likely be small relative to unlicensed devices designed to operate in unlicensed bands.

5.4 The Throughput-Coverage Product for the Unlicensed Devices

Clearly, there are a number of tradeoffs embedded in the relationships developed here. To show more clearly what they are, (85) can be written as:

$$L(d_u) = L(d_0) \cdot \frac{W_u a}{R_u K_u \Gamma_u} \tag{96}$$

Assuming that free space propagation applies to both d_0 and d_u,

$$\left(\frac{d_u}{d_0} \right)^2 = \frac{W_u a}{R_u K_u \Gamma_u} \tag{97}$$

Defining the service area of the unlicensed receiver as

$$A_u = p d_u^2, \tag{98}$$

(97) becomes

$$A_u \cdot \frac{R_u K_u}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{99}$$

but from (88), $pd_0^2 \cong P_{out} \frac{A_{cell}}{K_h}$, hence:

$$A_u \cdot \frac{R_u K_u}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{100}$$

Letting $R_{u,tot} = R_u K_u$ represent the total aggregate (raw) throughput of all the unlicensed devices,

$$A_u \frac{R_{u,tot}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{101}$$

Note that given a constant cell area K_u , $1/b_k$, *a*, and a bandwidth $A_{u,k} \cdot \frac{R_{u,k}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} b_k$ available to the unlicensed devices, the right hand side of (is constant and therefore $A_u R_{u,tot}$ is constant. This is a useful measure of the capacity available to the unlicensed devices, as rate and coverage can be traded off against each other.

Now suppose that not all of the unlicensed devices use the same rate or the same fraction of the interference budget $I_{\max}$. Let $b_k I_{\max}$ be the interference power received at the CDMA base station from the kth unlicensed transmitter, where:

$$\sum_{k=1}^{K_u} b_k = 1 \tag{102}$$

In the relationships derived above, K_u is replaced by $1/b_k$, and (99) becomes:

$$A_{u,k} \cdot \frac{R_{u,k}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} b_k \tag{103}$$

Summing over *k* gives:

$$\frac{\sum_{k=1}^{K_u} A_{u,k} R_{u,k}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{104}$$

which confirms that service area multiplied by data rate is an appropriate capacity measure. That is, all other factors being equal, $\sum A_{u,k} R_{u,k}$ is constant.

The above assumes that free space path loss applies to all of the distances involved. More generally, the smooth earth path loss can be written as:

$$L(d) = \begin{cases} \left(\frac{d}{d_{brk}}\right)^2 L(d_{brk}) & d \le d_{brk} \\ \left(\frac{d}{d_{brk}}\right)^4 L(d_{brk}) & d \ge d_{brk} \end{cases} \tag{105}$$

Suppose that $d_0 > d_{brk}$ and $d_u < d_{brk}$. Then

$$A_u \frac{R_{u,tot}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \frac{a}{\Gamma_u} \cdot \left(\frac{d_0}{d_{brk}}\right)^2 \tag{106}$$

(becomes:

$$A_u \frac{R_{u,tot}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \frac{a}{\Gamma_u} \cdot \left(\frac{d_0}{d_{brk}}\right)^2 \tag{107}$$

which simply increases the capacity of the unlicensed devices by the factor $(d_0/d_{brk})^2$.

If $d_u > d_{brk}$, then

$$\frac{L(d_u)}{L(d_0)} = \begin{cases} \frac{d_u^4}{(d_0 d_{brk})^2} & d_0 \le d_{brk} \\ \left(\frac{d_u}{d_0}\right)^4 & d_0 \ge d_{brk} \end{cases} \tag{108}$$

$$A_u^2 \frac{R_u K_u}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \frac{a}{\Gamma_u} \cdot \left(\frac{d_0}{d_{brk}} \right)^2 A_1 \qquad (109)$$

where

$$A_1 = \begin{cases} p d_{brk}^2 & d_0 \le d_{brk} \\ p d_0^2 & d_0 \ge d_{brk} \end{cases} \qquad (110)$$

In either case, the right hand side of (109) is constant, and A_u varies as $1 / \sqrt{R_{u,tot}}$, so reducing rate to gain coverage becomes a less favorable tradeoff than it is in the case of $d_u \le d_{brk}$.

5.5 Licensed Cost vs. Unlicensed Benefit

To assess the attractiveness of ITemp as a means to facilitate spectral efficiency in a CDMA uplink, it is useful to assume a perfect implementation and compare the value of what is gained in unlicensed capacity to the value of what is lost due to degradation of the licensed incumbent system. To make such a comparison, some measure of value is needed. As shown below, a spectrum efficiency equation that accounts for both throughput and coverage is a reasonable value measure.

5.5.1 Mobile and Portable Communication Spectrum Efficiency

Since the purpose of a mobile system is to provide wireless throughput over some service area, it seems reasonable that the value from the user's perspective would be related to the data rate and the extent of the wireless coverage. Further, value should vary linearly with data rate; if a rate of R_1 can provide a voice circuit for a single subscriber, a rate of $2R_1$ can serve two subscribers and is therefore twice as valuable, delivering twice the revenue to the service operator. As will be seen, the cost in terms of both spectrum and infrastructure is also proportional to rate.

Coverage should also be a factor in the value measure, because the more complete the coverage, the greater the value to the subscriber. Clearly, a wireless channel that is available over a complete metropolitan area is more valuable to the customer than one limited to a home, an office building, or selected "hotspots."

To develop a reasonable measure, consider first a CMRS system which uses an air interface that delivers a spectrum efficiency of h_{AI} bps/MHz/cell. In the context of this discussion, a "cell" is the coverage area of a single base station antenna/transceiver system, and may be a sector in actual implementation. However, with respect to coverage and capacity, a sector is simply a cell with a “pie slice” shape.

If the bandwidth per direction (uplink/downlink) available to the system is B, then the throughput per cell is

$$R_{cell} = \eta_{AI} B \text{ bps/cell} \tag{111}$$

Over some service area A_{svc} there are N_{cell} cells, and the total throughput provided by the system over its service area is

$$R_{svc} = R_{cell} N_{cell} = \eta_{AI} B N_{cell} \text{ bps} \tag{112}$$

Assuming coverage is provided over the entire service area, R_{svc} is a potential value measure, since both infrastructure and spectrum costs, as well as revenue, vary in proportion to R_{svc}. To increase R_{svc}, an operator would need to either add more cells or obtain more spectrum, both of which have associated costs. For example, R_{svc} could be doubled by either doubling *B* or doubling N_{cell} (or by lesser increases in both *B* and N_{cell}).

If coverage is provided only over some fraction of the service area A_{svc} (i.e., the total area with actual coverage is A_{cvr}), then the service is more limited and is of less value. For example, a network of “hotspots” with cell density ρ_{cell} and total throughput R_{svc} but very localized coverage (e.g., $A_{cvr}/A_{svc} = 0.01$) is not as valuable to subscribers as a total-coverage service with the same total throughput. The value measure therefore must account for the degree of coverage provided. Multiplying (112) by the fractional coverage defined as $f_{cvr} \equiv A_{cvr}/A_{svc}$ gives a better measure of value as

$$V = R_{svc} f_{cvr} = \eta_{AI} f_{cvr} B N_{cell} \tag{113}$$

Since R_{svc} scales with *B* and N_{cell}, a reasonable value measure for cost/benefit analysis of ITemp is the spectrum efficiency defined as

$$\eta = \eta_{AI} f_{cvr} \tag{114}$$

and the total value for a network is simply

$$V = h BN_{cell} \tag{115}$$

As shown in section 6 below, there is a tradeoff between cell coverage area and cell throughput (given the available bandwidth). That is, h_{AI} can be traded off for f_{cvr}, so for a given investment in infrastructure (a given cost), coverage and throughput can be traded off. That is why the value measure used here must account for both throughput and coverage; otherwise, the "value" could be artificially increased by raising throughput at the expense of coverage.

5.5.2 ITemp Cost/Benefit Analysis for CMRS

From the previous section, the throughput per sector lost to the CDMA uplink is

$$\Delta R_{TOT} = \frac{\Delta T/T}{\Phi(1+m)} \cdot R \cdot (M+1) \tag{116}$$

so the loss in value per sector for the licensed CMRS system is

$$\Delta h_l = \frac{\Delta R_{tot}}{W} = \frac{\Delta T/T}{\Phi(1+m)} \cdot \frac{R}{W} \cdot (M+1) \tag{117}$$

In this case, full coverage is assumed so that $f_{cvr} = 1$.

From (104),

$$\frac{\sum_{k=1}^{K_u} A_{u,k} R_{u,k}}{W_u} \cong P_{out} \cdot \frac{A_{cell}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{118}$$

If N_{sctr} is the number of sectors in a cell and A_{sctr} is the service area per sector, then $A_{cell} = N_{sctr} A_{sctr}$. The fractional coverage for the k^{th} unlicensed link is $A_{u,k}/A_{sctr}$ and its value measure can then be expressed as

$$h_{u,k} = \frac{A_{u,k}}{A_{sctr}} \frac{R_{u,k}}{W_u}. \tag{119}$$

The total value of the unlicensed capacity that is gained is

$$h_u = \sum_{k=1}^{K_u} h_{u,k} = \frac{\sum_{k=1}^{K_u} A_{u,k} R_{u,k}}{A_{sctr} W_u} \cong \frac{N_{sctr} P_{out}}{K_h} \cdot \frac{a}{\Gamma_u} \tag{120}$$
$$= \frac{P_{out}}{K_{h,sctr}} \cdot \frac{a}{\Gamma_u}$$

where $K_{h,sctr} = K_h / N_{sctr}$ is the number of active CDMA handsets per sector.

With

$$a = \frac{I_{\max}}{I_{TOT}} (M+1) \frac{L_u}{L_h} \frac{1}{ICF_{u,c} \cdot ICF_{c,u}} . \tag{121}$$

With $I_{\max} / I_{TOT} = (\Delta T / T) / \Phi$, and letting $L_u = L_h$, and $ICF_{u,c} = 1$ and $ICF_{c,u} = 1$ (the unlicensed and CMRS signals affect each other in the same way as noise),

$$a = \frac{\Delta T / T}{\Phi} (M+1) \tag{122}$$

$$\frac{h_u}{\Delta h_l} = P_{out} \frac{1+m}{K_{h,sctr} \Gamma_u} \frac{W}{R} = P_{out} \frac{M}{K_{h,sctr}} \frac{\Gamma}{\Gamma_u} (1+m) \tag{123}$$

Taking the ratio of (117) and 122) gives

$$\frac{h_u}{\Delta h_l} = P_{out} \frac{1+m}{K_{h,sctr} \Gamma_u} \frac{W}{R} = P_{out} \frac{M}{K_{h,sctr}} \frac{\Gamma}{\Gamma_u} (1+m) \tag{124}$$

The number of handsets that can be supported per sector clearly varies with the jamming margin, and for IS-95 and 1XRTT a reasonable approximation is:

$$K_{h,sctr} \cong \frac{M}{2} \tag{125}$$

Letting $m = 0.5$, (123) becomes:

$$\frac{h_u}{\Delta h_l} \cong 3P_{out} \frac{\Gamma}{\Gamma_u} \tag{126}$$

This suggests that the value gained in terms of unlicensed throughput-coverage is substantially less than the value given up due to reduced licensed capacity. For example, assume that $\Gamma_u = 10\,\text{dB}$, $\Gamma = 7\,\text{dB}$, and $P_{out} = 0.05$. In that case, $h_u / \Delta h_l = 0.075$. Moreover, as the CMRS technology improves, this ratio will decrease. For example, if Γ is reduced by 3 dB (consistent with 1xRTT compared to IS-95), then $h_u / \Delta h_l$ is reduced by a factor of 2.

It is perhaps more meaningful to show the total throughput-coverage as a percentage of the original throughput-coverage of a CMRS system operating in unshared spectrum, using

$$\Delta h = \Delta h_l - h_u = \frac{\Delta T / T}{\Phi}(M+1)\left[\frac{R}{W} \cdot \frac{1}{1+m} - \frac{P_{out}}{K_{h,sctr}} \cdot \frac{1}{\Gamma_u}\right] \tag{127}$$

The value of the undisturbed CDMA system is

$$h_0 = \frac{R_{TOT}}{W} = \frac{R}{W} \Lambda_0 (M+1) \tag{128}$$

Figure 26 shows $1 - \Delta h / h_0$ (in percent) versus $\Delta T / T$ (in dB) for 1xRTT CDMA CMRS system for different unlicensed outage probabilities. As can be seen, the net change in overall efficiency is negative.

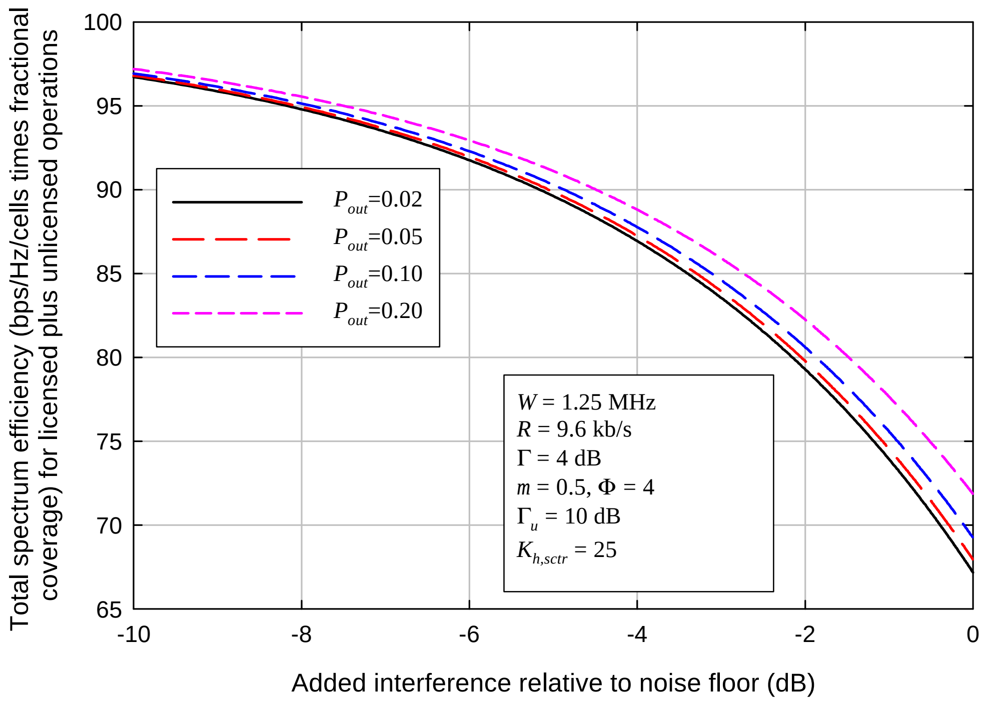

Figure 26: *Effect of ITemp sharing on total spectrum efficiency, including the added unlicensed capacity, for a CDMA uplink.*

5.6 Conclusions

The analysis above assumes that the ITemp implementation in the CDMA uplink is perfect – that is, there is some monitoring and feedback mechanism at the CRMS base station that perfectly tracks and controls the total interference added to the CMRS uplink band by the unlicensed devices. Although it is far from clear how this could actually be realized, it is useful to ignore implementation problems in order to explore the ultimate theoretical potential of spectrum sharing using ITemp, taking into account both the impact on the licensed system and the benefits gained from operation of the unlicensed devices.

Using an efficiency measure that is the product of throughput and the fractional coverage, it is clear that the loss in value to the licensed service is greater than added value associated with the unlicensed devices. This is not completely surprising, because the scenario analyzed here constitutes a mixing of unlike radio systems, which often causes inefficiencies.

6. The Throughput vs. Coverage Tradeoff

6.1 Introduction

The purpose of this section is to explain the inherent throughput vs. coverage tradeoff that exists for wireless mobile communications and to underscore the point that assessment of the "value" of a wireless network (or system) cannot be based on throughput alone. With a given physical topology (*e.g.*, base station density) and quantity of spectrum, throughput can be increased by sacrificing coverage. Because of this inherent tradeoff, the value of a wireless system should be based on some combination of throughput and the fraction of the nominal service area that is actually covered. The results below suggest that a reasonable value measure is the product of throughput and fractional coverage.

The throughput/coverage tradeoff is a consequence of two relationships. The first is captured by Figure 27, which shows the achievable modulation efficiency or spectral efficiency R/W (bps/Hz) vs. the signal-to-interference plus noise ratio (SINR) at the receiver (it is assumed that the interference affects the receiver in the same manner as noise of the same power level). The relationship is shown for some classical (uncoded) modulation formats, as well as the Shannon bound. Also shown are points for the downlink of the 3G wireless data standard 1xEvDO (from [1]), which are closer to the Shannon bound due to the use of coding.

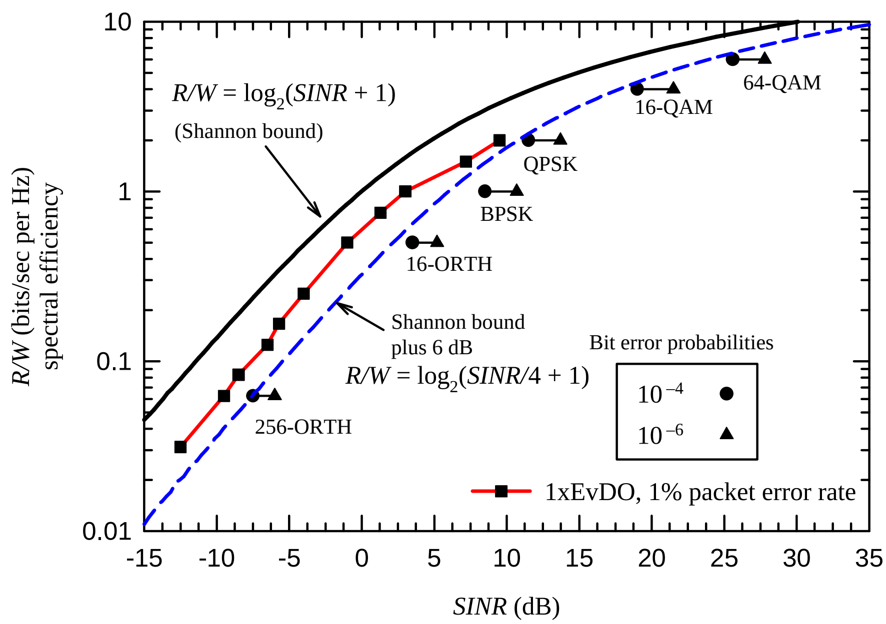

Figure 27: *Modulation efficiency vs. signal to interference plus noise ratio (SINR)*

Note that at a bit error rate (BER) of 10^{-4}, most of the classical formats shown roughly track a curve offset from the Shannon bound by about 6 dB; that is,

$$\frac{R}{W} \cong \log_2\left(SINR/d + 1\right) \tag{129}$$

where $10\log d$ represents the offset from the Shannon bound in dB.

The second relationship is simply the fact that for a given transmitter location and transmitted signal power, the received signal power decreases as the transmitter-receiver distance increases. For most propagation environments of interest, the relationship between the transmitted power P_{tx}, the received power P_{rx}, and the transmitter-receiver distance *d* can be expressed as:

$$P_{rx} = aP_{tx}d^{-g} \tag{130}$$

where a is a constant that depends on the frequency antenna characteristics, and g is the path loss exponent, typically between 3 and 4 for mobile propagation.

Assuming for the moment that the noise plus interference ($N + I$) is fixed, then (129) and (130) can be combined to give:

$$R \cong W \log_2\left(kP_{tx}d^{-g} + 1\right) \tag{131}$$

where $k = \dfrac{a}{d\left(N+I\right)}$. The throughput vs. coverage tradeoff arises because larger coverage corresponds to larger values of *d*, and therefore lower average rates.

6.2 Average Throughput over a Coverage Area

Using (130), the average rate over a circular coverage area with radius r and a transmitter at the center is

$$\overline{R}_1 = W\int_0^r \log_2\left(kP_{tx}x^{-g} + 1\right)\cdot f_d\left(x\right)dx \tag{132}$$

where $f_d\left(x\right)$ is the probability density function (PDF) of the distance from the base station. For a uniform distribution of receiver positions over a circular coverage area,

$$f_d\left(x\right) = \frac{2x}{r^2} \qquad 0 \le x \le r \tag{133}$$

In a multiple access context, the linear averaging of the rate per (132) represents a resource allocation policy under which equal time (or equal total transmit energy) is devoted to each mobile within distance *r* of the base station, regardless of the rate at which it can receive.

Another potential policy is that of equal average rates – that is, the length of the message that must be received is independent of location, so more transmission time is required for locations at which the rate is low. In that case, it is the time required to transmit a bit that must be averaged rather than the rate itself, giving an average rate of:

$$\overline{R}_2 = \langle T_b \rangle_d^{-1} = \left\langle \frac{1}{R} \right\rangle_d^{-1} = W \left\{ \int_0^r \left[\log_2 \left(kP_{tx} x^{-g} + 1 \right) \right]^{-1} \cdot f_d(x) dx \right\}^{-1} \tag{134}$$

where $\langle \cdot \rangle_d$ denotes averaging over distance.

To illustrate, assume that at some distance $r_{\max}$ the SINR is Γ. Then

$$R \cong W \log_2 \left(\frac{\Gamma}{d} (d/r_{\max})^{-g} + 1 \right) \tag{135}$$

Letting $s = d/r_{\max}$, the PDF of s is $f_s(x) = 2x/(r/r_{\max})^2$, $0 \le x \le r/r_{\max}$, and (133) and (134) become, respectively:

$$\overline{R}_1 = W \int_0^{r/r_{\max}} \log_2 \left(\Gamma x^{-g}/d + 1 \right) \cdot f_s(x) dx \tag{136}$$

$$\overline{R}_2 = W \left\{ \int_0^{r/r_{\max}} \left[\log_2 \left(\Gamma x^{-g}/d + 1 \right) \right]^{-1} \cdot f_s(x) dx \right\}^{-1} \tag{137}$$

Figure 28 shows the results for $\Gamma = 1/\sqrt{10}$ (–5 dB), $d = 4$ (6 dB), and $g = 3.5$. The abscissa is the fractional coverage

$$f_{cvr} = (r/r_{\max})^2 \tag{138}$$

The reference line is $0.6/f_{cvr}$, placed to show that for the average throughput varies approximately inversely with the fractional coverage for most of the cell (the factor of 0.6 was chosen simply for useful placement of the reference line).

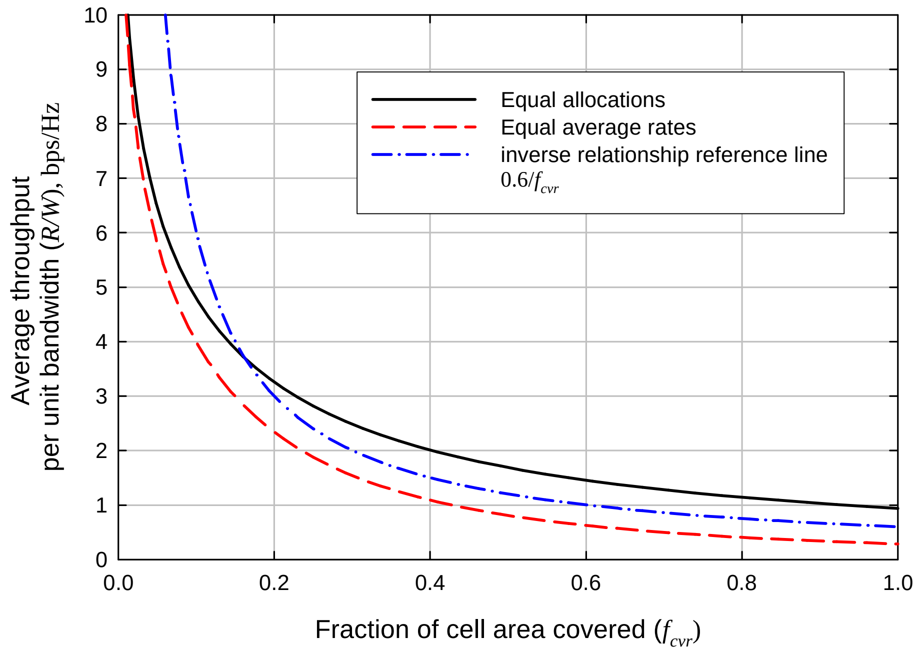

Figure 28: *Illustration of the throughput vs. coverage tradeoff*

While Figure 28 serves to illustrate the throughput vs. coverage tradeoff, the analysis was somewhat idealized, as it assumed constant $N+I$, and assumed a continuum of modulation rates. In an actual frequency reuse system such as a cellular network, the interference I is self-interference (signals from other cells), which varies with d (although mildly), and there will be a finite number of discrete modulations efficiencies from which to choose. The next subsection incorporates these factors into the analysis.

6.3 Cellular System Analysis with Discrete Modulation Rates

The layout of base transmitters is modeled as a regular hexagonal grid. In the absence of fading, the actual cell boundaries (based on signal strength) are also hexagonal as shown in Figure 29. The nominal cell radius (center to vertex) is denoted r as shown.

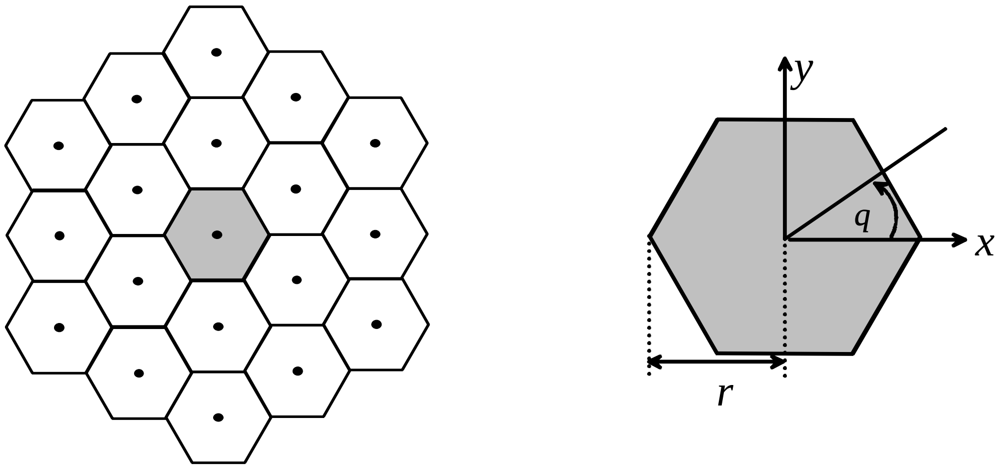

Figure 29: *Assumed cell geometry for computing cochannel interference (2 tiers of interfering cells).*

This analysis will focus on the downlink, and thermal noise is assumed negligible compared to cochannel interference, which is the sum of contributions from neighboring downlink transmitters as shown in Figure 29 (this case would apply if all transmitters use the same frequency). For the non-faded case, the signal-to-interference ratio depends on the distance of a mobile from the cell center, as shown in Figure 30. The desired signal power is simply $S(x) = x^{-g}$, and the outer cell interference approximation for $q = 0$ is

$$I(x) \cong 2\left[\left(\sqrt{3}+x\right)^{-g} + \left(3+x^2\right)^{-g/2} + \left(3-3x+x^2\right)^{-g/2}\right] + 6\left[3^{-g} + \left(2\sqrt{3}\right)^{-g}\right] \qquad \textbf{(}139\textbf{)}$$

The first sum (3 terms) represents interference from the first tier (the six cells immediately adjacent to the cell of interest) and the other two terms approximate the interference from the second tier (12 cells). Note that the interference from the second tier is approximated as independent of *x*. As can be seen from Figure 30, the approximation is extremely close to the exact *S/I* for $q = 0$. For $q > 0$, *S/I* is slightly lower than it is for $q = 0$, as can be seen in the expanded view of Figure 30 for the extreme case $q = 30^\circ$. The exact *S/I* will be used in the simulations and the approximation of **(**139**)** will be used in the analyses.

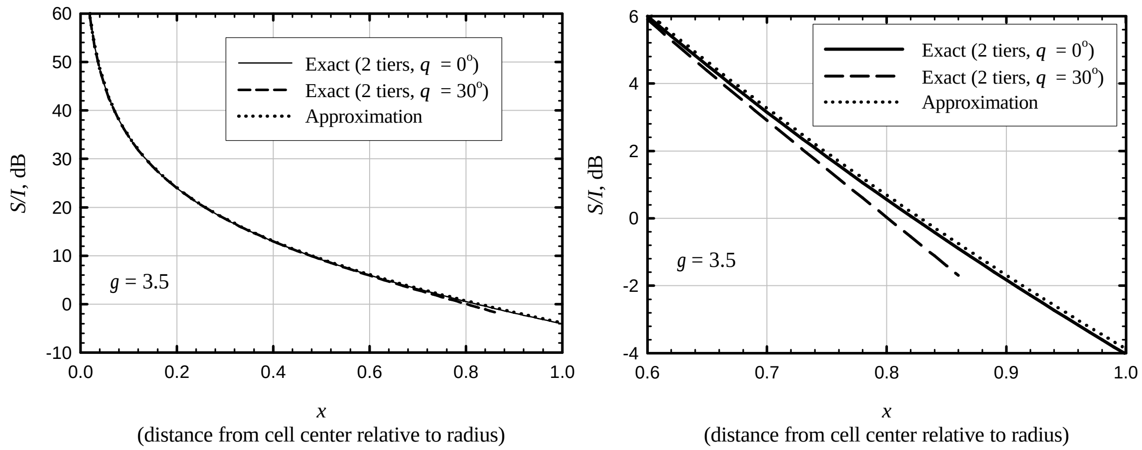

Figure 30: *S/I as a function of mobile location; full-scale and expanded.*

The relationship between the spectral efficiency *R/W* and the signal-to-interference ratio *S/I* that will be used is:

$$\frac{R}{W} = F_q\left[\log_2\left(\frac{S/I}{d}+1\right)\right] \tag{140}$$

where as discussed above, $\Delta = 10\log d$ is the offset from the Shannon bound in dB, and will depend on the required error rate, the coding used, and the receiver detection performance. $F_q[\cdot]$ is a quantization function, since by assumption *R/W* can assume only a limited set of discrete values.

The quantization assumed here is:

$$F_q(n) = \begin{cases} \lfloor n \rfloor, & n \geq 1 \\ (k+1)/2^k, & \ni\ (k+1)/2^k \leq n \leq k/2^{k-1}, \quad k > 1, n < 1 \end{cases} \tag{141}$$

Thus, permitted rates are: *R/W* = . . . 4, 3, 2, 1, 3/4, 1/2, 5/16, etc., subject to some absolute maximum and minimum, which will be treated as parameters for this analysis. The graph in Figure 31 shows both continuously-variable and quantized spectral efficiencies vs. normalized distance of the mobile from the cell center, and the illustration shows a conceptual example of spectral efficiency contours.

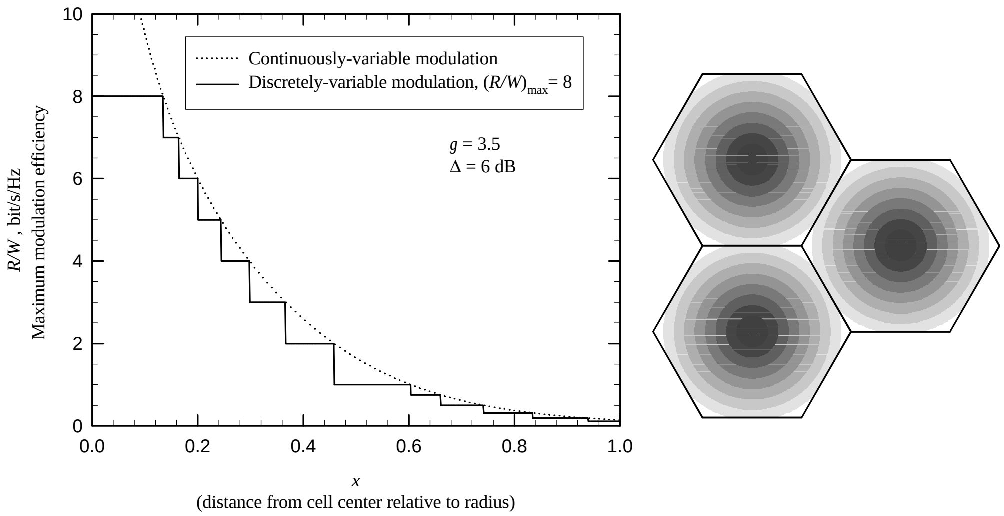

Figure 31: *Spectral efficiency vs. distance of mobile from cell center as a fraction of cell radius and conceptual illustration of spectral efficiency contours (the darker the shading, the higher the rate).*

6.4 Average Downlink Throughput per Cell

It is assumed here that the rate used for each transmission will be the highest possible, as dictated by **(**140**)**. It is assumed for the present that served mobiles are uniformly-distributed over area (i.e., there is no bias against low *S/I* mobiles which are more costly to serve). For a hexagonal coverage area of unit radius, the probability density function (PDF) of the distance from the center as a fraction of the hexagon radius is:

$$f_x(x) = \begin{cases} \dfrac{4px}{3\sqrt{3}} & x \le \dfrac{\sqrt{3}}{2} \\ \dfrac{8x}{\sqrt{3}}\left(\dfrac{p}{6} - \cos^{-1}\dfrac{\sqrt{3}}{2x}\right) & \dfrac{\sqrt{3}}{2} \le x \le 1 \end{cases} \tag{142}$$

and the associated cumulative distribution function, which is important in generating random mobile locations in simulations, is:

$$\Pr(x < x) = \begin{cases} \dfrac{2px^2}{3\sqrt{3}} & x \le \dfrac{\sqrt{3}}{2} \\ \dfrac{4x^2}{\sqrt{3}}\left(\dfrac{p}{6} - \cos^{-1}\dfrac{\sqrt{3}}{2x}\right) + \sqrt{4x^2 - 3} & \dfrac{\sqrt{3}}{2} \le x \le 1 \end{cases} \tag{143}$$

Figure 32 shows $f_x(x)$ and $\Pr(x < x)$. A circular coverage area has a much simpler PDF: $f_x(x) = 2x/x_{\max}^2$, $0 \le x \le x_{\max}$. However, dividing the plane into non-overlapping hexagonal areas for simulation purposes imposes a symmetry that in some cases allows results from a single hexagonal area to be taken as representative.

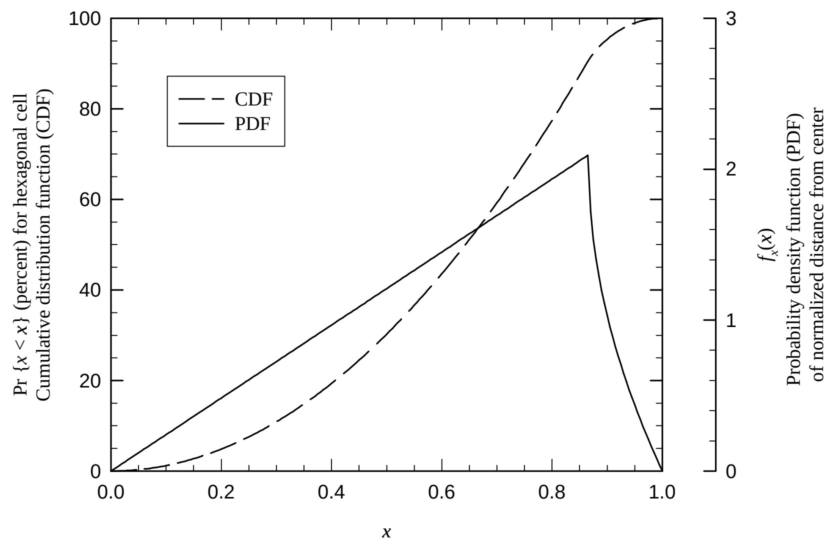

Figure 32: *PDF and CDF of distance from the center of a hexagonal coverage area.*

If the bit rate for a particular mobile is R b/s, the time required to transmit one bit is $1/R$, so the time required to transmit one bit, averaged over mobiles uniformly distributed in a hexagonal coverage area, is:

$$\langle T_b \rangle = \left\langle \frac{1}{R} \right\rangle = \frac{1}{W} \left\langle \frac{1}{R/W} \right\rangle \cong \frac{1}{W} \int_0^X \left\{ F_q \left[\log_2 \left(\frac{1}{d} \frac{S(x)}{I(x)} + 1 \right) \right] \right\}^{-1} f_x(x) dx \,, \quad X \le 1 \qquad \textbf{(144)}$$

The average capacity in bits/sec per cell or sector is simply $1/\langle T_b \rangle$ bps and the average spectral efficiency is $1/W\langle T_b \rangle$ bps/Hz. The formula for $\langle T_b \rangle$ is an approximation because **(**139**)** is used for $I(x)$, which does not account for the effect of azimuth angle q on the signal-to-interference ratio.

Restricting coverage to mobiles near the base station effectively places a lower bound on the rate that can be used, and gives a higher average throughput than if all mobiles are served regardless of location. Figure 33 shows the average spectral efficiency per cell vs. the fraction of the cell area covered, which corresponds to X^2, where X is the upper limit of the integral in **(**144**)**. The average was evaluated using both the analytic approach of

(144**)** with the integral evaluated numerically, and by a Monte Carlo simulation.[7] Also, both hexagonal and circular coverage areas were explored. The circle radius was $\sqrt{2.6/p}$ times the hexagon radius to make the total areas equal. As can be seen, agreement is excellent among the four cases. Also shown is a reference line for which the modulation efficiency in inversely proportional to the fractional coverage.

Figure 34 shows the effect of the upper limit on the modulation rate. As might be expected, the high rates are beneficial primarily when coverage is limited to a small fraction of the cell area.

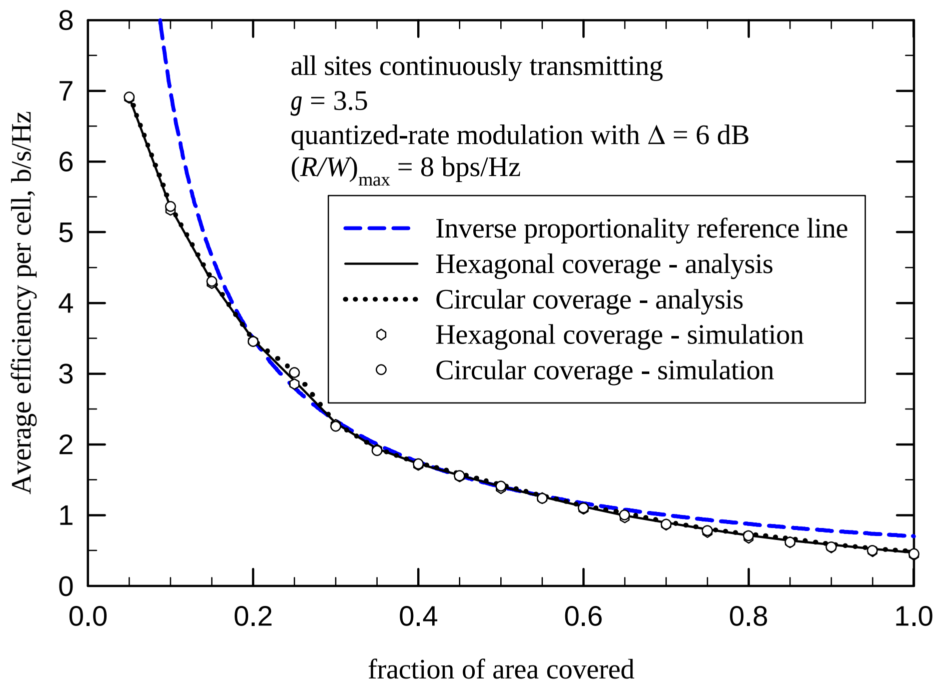

Figure 33: *Average spectral efficiency vs. coverage.*

[7] The Monte Carlo simulation positions mobiles uniformly over a 60-degree cell sector and calculates the outer cell interference exactly rather than using the approximation of (3).

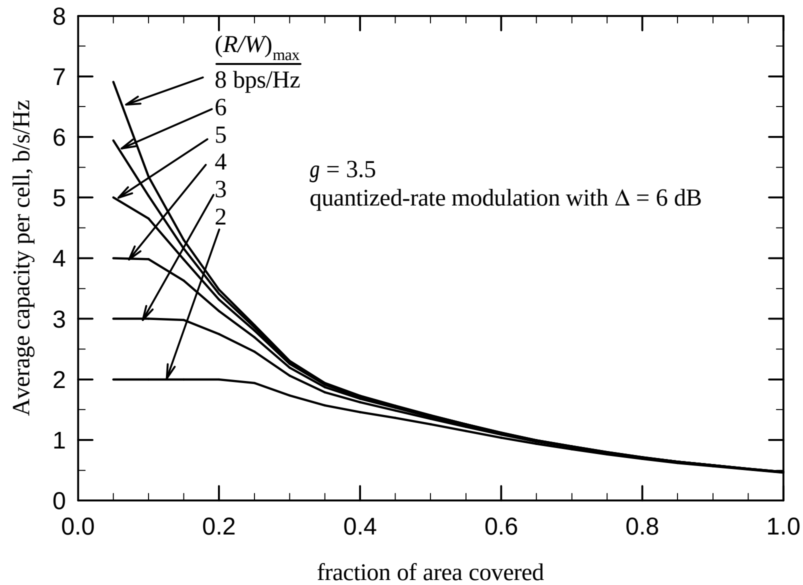

Figure 34: Effect of upper modulation rate limit on average capacity.

6.5 Outage, Minimum Rate, and Shadow Fading

From Figure 31 it is clear that the setting of a minimum rate is equivalent to excluding mobiles near the outer edge of the cell. Without shadow fading, a given minimum rate corresponds to a maximum distance from the base station at which the mobile can still receive service. With shadow fading, this is not the case.

The effect of shadow fading was incorporated into the Monte Carlo routine. Shadow fading was assumed to be the combined effect of two components: a local component associated with the mobile, which has the same effect on the path loss to each base station, and thus cancels out in the signal-to-interference calculation, and a separate base-dependent component. The dB value of each shadow fading component (both assumed lognormal) has a standard deviation of $s/\sqrt{2}$, so the sum of the base-dependent and local components (in dB) has a standard deviation of s. This is the same approach taken by Viterbi [2] (chapter 6) for analyzing the capacity of a CDMA cellular system.

In the Monte Carlo procedure, a minimum rate $(R/W)_{\min}$ is set. For each sample mobile location, *S/I* is calculated, including the effects of shadow fading. It is assumed that the mobile will choose the base transmitter that delivers the highest *S/I*. Based on the *S/I*, the maximum achievable transmission rate is computed. If this maximum is less than $(R/W)_{\min}$, the mobile experiences an outage. The fraction of mobiles experiencing outage (P_{out}) is accumulated over all samples, as is the average rate for the mobiles that could be served (calculated by averaging the inverse of the rate, then inverting the result as before). This procedure is repeated for a range of values of $(R/W)_{\min}$, and average rate is plotted against $1-P_{out}$, which is the fraction of mobiles that receive service. This corresponds to the fraction of the cell area over which service is available, although it does not have the same geometric interpretation as in the non-faded case studied earlier. Figure 35 shows the result. As can be seen, the mathematical model still provides a reasonable approximation despite the fact that it does not account for shadow fading.

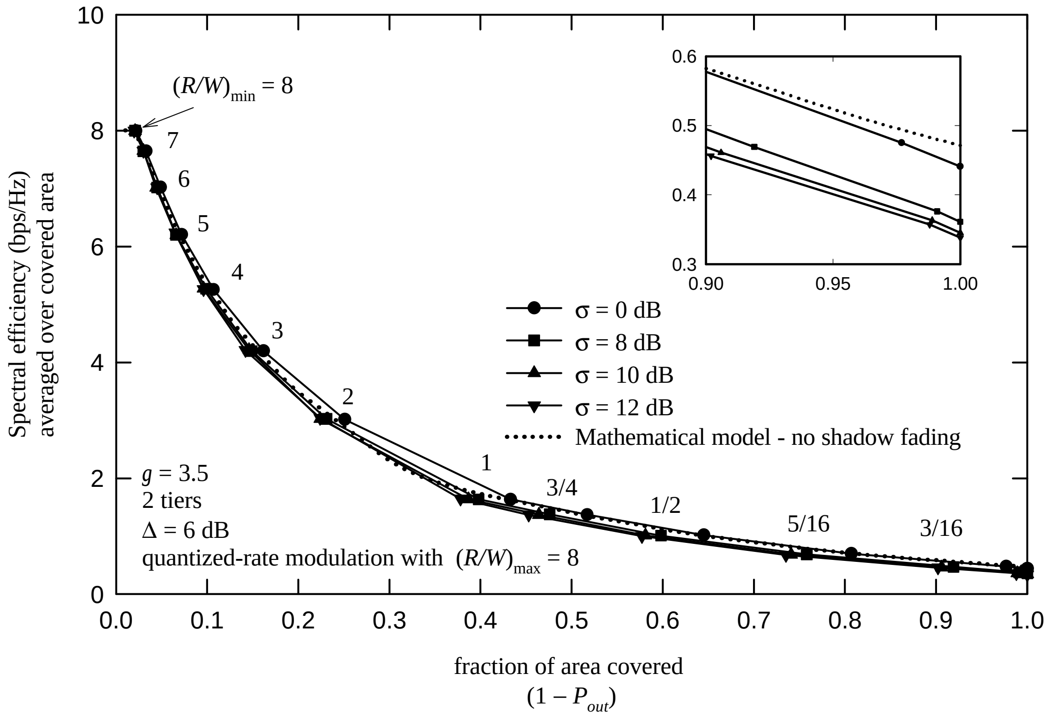

Figure 35: *Spectral efficiency vs. coverage with shadow fading.*

6.6 Summary

This section has explained the throughput vs. coverage tradeoff quantitatively, and has demonstrated that tradeoff with some examples. Clearly, the exact relationship between coverage and throughput in a given case will depend on specific factors such as the resource allocation policy, the propagation parameters, and the range of modulation and coding rates available to the air interface. However, the results given here suggest that, at least for terrestrial mobile systems, the achievable throughput varies roughly as the inverse of the fractional coverage (area actually covered relative to the nominal service area).

In other words, with modern radio technology that can adjust the transmission rate to quality of the radio link, coverage (range) can be sacrificed to achieve greater throughput, or vice versa. That is why throughput multiplied by fractional coverage is an appropriate measure of spectrum efficiency.

Section 6 References

[1] P. Bender, *et. al.*, "CDMA/HDR: A Bandwidth-Efficient High-Speed Wireless Data Service for Nomadic Users," *IEEE Commun. Mag.*, pp. 70-77, July 2000.

[2] A. J. Viterbi, *CDMA*: *Principles of Spread Spectrum Communication*. Reading, MA: Addison-Wesley, 1995.